Sigmoidal Functions of the Forms

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$g(x) = \frac{2}{1 + e^{-\alpha x}} - 1$$

$$h(x) = \tanh(\alpha x)$$

where α is a positive parameter

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NEURAL NETWORKS

(1) Consider the binary sigmoidal function

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

Verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{f(x)}{1 - f(x)} \right]$$

(2) A neuron receives inputs 0.5, 1.5, and −1.1 with weights 0.7, 0.9, and 1.2, respectively. If the neuron produces an output signal s of the sigmoidal form

$$s = \frac{1}{1 + e^{-0.6y}}$$

where y is the activation, find the values of y and s. Take the bias weight as 1.3.

(3) A neuron receives inputs 0.6, 1.7, and −1.5 with weights 0.6, 1.1, and 1.3, respectively. It employs a sigmoidal function of the form

$$f(y) = \frac{1}{1 + e^{-\alpha y}}$$

where y is the activation and α is a positive parameter. Find the value of α such that the output signal is 0.66. Take the bias weight as 1.1.

(4) A neuron produces a sigmoidal signal of the form

$$s = \frac{1}{1 + e^{-\alpha y}}$$

where y is the activation and α is a positive parameter. The inputs to the neuron are $x_1 = -0.9$, $x_2 = 0.9$, and $x_3 = 1.2$ with respective weights $w_1 = 0.8$, $w_2 = -0.8$, and $w_3 = 0.4$. Under certain operating conditions, the output signal s is found to be 0.5. Find the value of the bias weight w_0. Comment on the (corresponding) value of α.

(5) For the binary sigmoidal function

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

$$\frac{df(x)}{dx} = \frac{\alpha e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

or, alternatively,

$$\frac{df(x)}{dx} = \alpha f(x)[1 - f(x)]$$

(6) Sketch the graph of $\frac{df(x)}{dx}$ vs. $f(x)$ for $\alpha = 0.5, 1,$ and 1.5. Show that the maximum value of $\frac{df(x)}{dx}$ is 0.25α and occurs at $f(x) = 0.5$.

(7) A neuron receives two inputs $x_1 = 1.5$ and $x_2 = 1.25$ with weights $w_1 = -1$ and $w_2 = 2$, respectively. The output signal s obeys a sigmoidal function of the form

$$s = \frac{1}{1 + e^{-2y}}$$

where y is the activation. Find the bias weight w_0

when the derivative of s with respect to y is 0.33. What is the corresponding value of s ?

(8) Consider the neural network illustrated in Fig. 1. The inputs are $x_1 = 2$ and $x_2 = -1.5$. The weights (including bias) are

$w_{13} = -1$	$w_{23} = 1.1$	$w_{35} = 1.1$
$w_{14} = -0.5$	$w_{24} = 1.2$	$w_{45} = -1.1$
$w_{03} = 0.7$	$w_{04} = -0.5$	$w_{05} = 0.9$

The two hidden neurons and the output neuron all employ sigmoidal functions of the form

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

with $\alpha = 0.8$ for each hidden neuron and $\alpha = 0.6$ for the output neuron. Find the value of the output signal s.

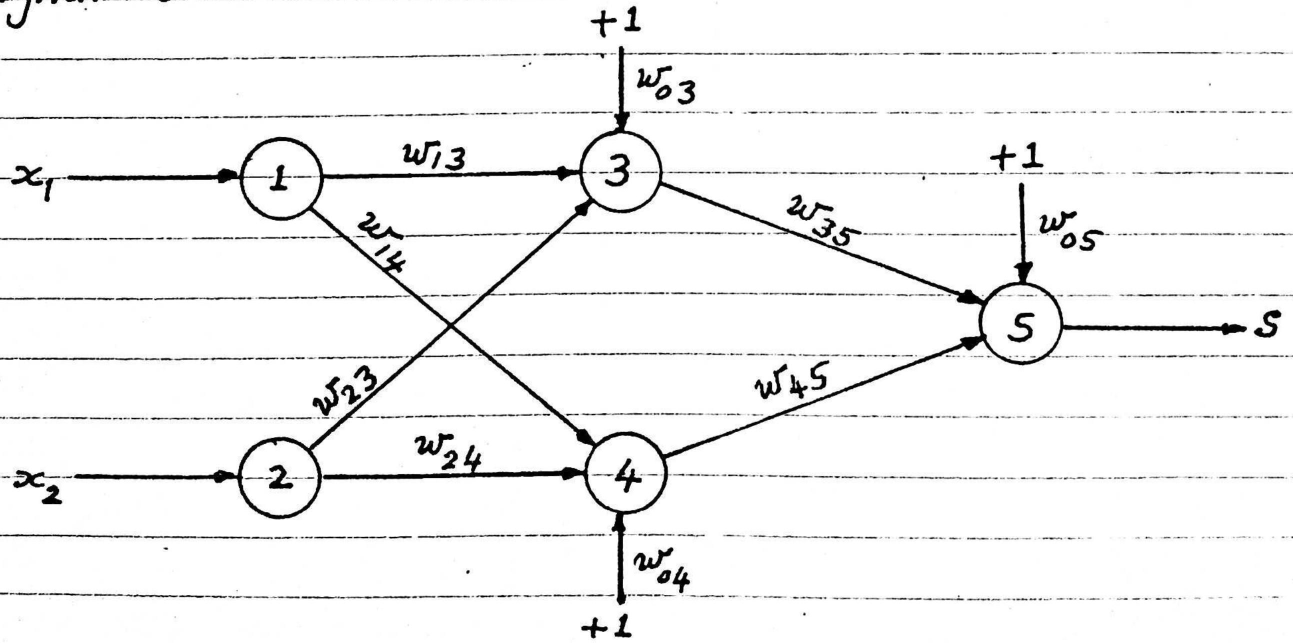

Fig. 1 Neural network for Prob. 8

(9) Consider the bipolar sigmoidal function

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

a) Draw, on the same coordinate axes, the graphs of $g(x)$ for $\alpha = 0.5$, 1, and 2. Comment on these graphs.

b) Verify that

$$x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]$$

c) Verify that

$$\frac{dg(x)}{dx} = 0.5\alpha \left[1 - g^2(x) \right]$$

(10) The neuron illustrated in Fig. 2 receives inputs $x_1 = 0.5$, $x_2 = 0.4$, $x_3 = 0.6$ with weights $w_1 = 1.1$, $w_2 = -2.1$, $w_3 = 0.5$ and the bias weight $w_0 = 1.7$. The output signal s is produced according to a sigmoidal function of the form

$$s = \frac{2}{1 + e^{-\alpha y}} - 1$$

where y is the activation. Find the value of the parameter α such that $s = 0.75$.

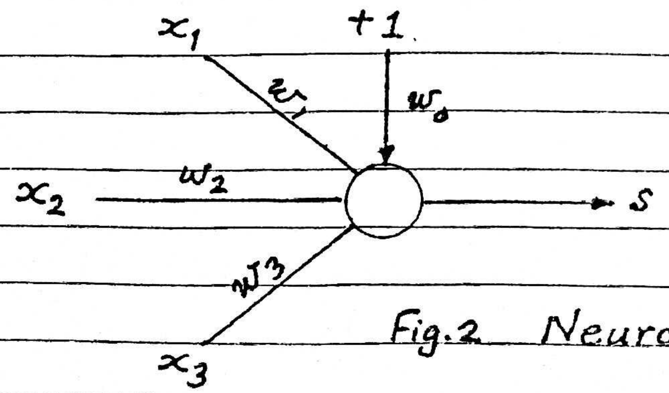

Fig.2 Neuron for Prob. 10

(11) In Prob. 10, with the value of α arrived at, let the bias weight w_0 be halved in value while all other weights and the inputs are kept unaltered. What is the new value of the output signal s ?

(12) In Prob. 11, find the value of the derivative of the output signal s with respect to the activation y.

(13) Show that the bipolar sigmoidal function

$$g(x) = \frac{2}{1+e^{-2x}} - 1$$

is the same as the hyperbolic tangent function $\tanh x$ and that this is a special case of the relationship

$$\frac{2}{1+e^{-\alpha x}} - 1 = \tanh\left(\frac{\alpha x}{2}\right)$$

(14) A neuron receives two inputs $x_1 = 0.7$ and $x_2 = 0.9$ with weights $w_1 = 1.5$ and $w_2 = -1.5$, respectively. The bias weight is $w_0 = 0.8$. If the neuron employs a hyperbolic tangent function, find the value of the output signal.

(15) In Prob. 14, the inputs x_1, x_2 and the weights w_1, w_2 are all kept unchanged while the bias weight w_0 is allowed to change. Find the value of w_0 if the

output signal is to be 0.81.

(16) In Prob. 15, find the value of the derivative of the output signal with respect to the activation.

(17) A neuron employs a hyperbolic tangent function. Under certain operating conditions, the derivative of the output signal s with respect to the activation y is found to be 0.441. Find the values of y and s.

(18) Consider the two-input, two-output neural network illustrated in Fig. 3. All hidden and output neurons employ hyperbolic tangent functions of the form

$$h(x) = \tanh(\alpha x)$$

with $\alpha = 0.5$ for each hidden neuron and $\alpha = 1.5$ for each output neuron. Find the values of the output signals s_1 and s_2.

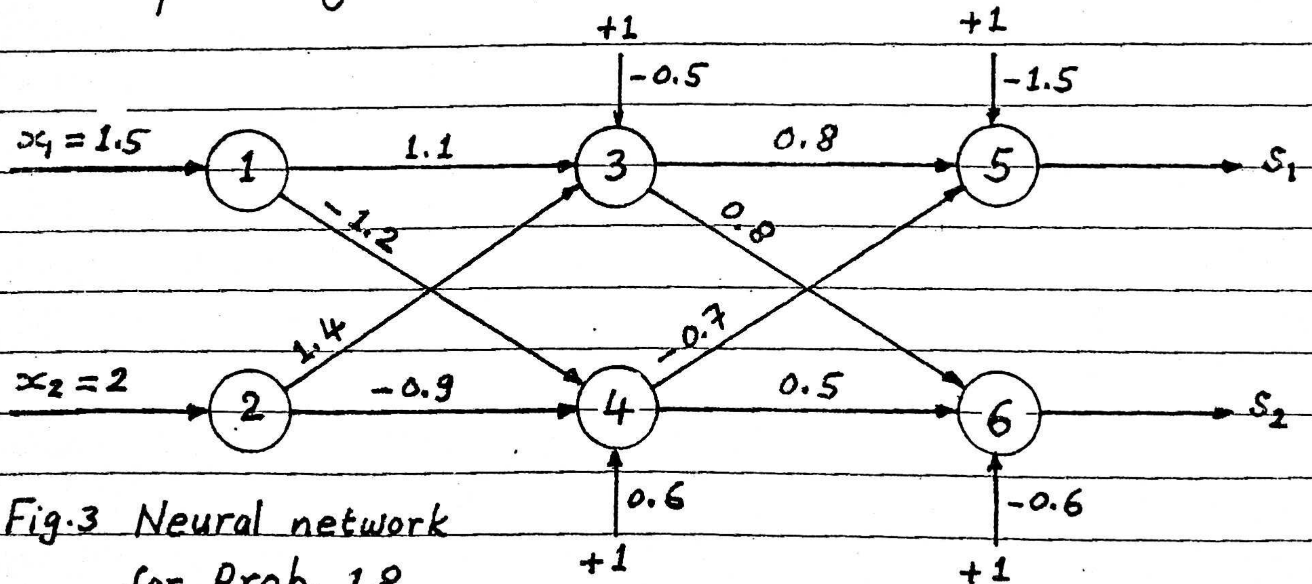

Fig. 3 Neural network for Prob. 18